AIAA 80-1808R

Simple Two-Dimensional-Nozzle Plume Model for Infrared Analysis

Chong-Wei Chu,* Joe Der Jr.,† and Wyman Wun‡
Northrop Corporation, Hawthorne, Calif.

A simple modeling technique for predicting detailed flow properties in a two-dimensional (2D)-nozzle plume exhausting into quiescent air is presented. The model is constructed by splitting a scaled axisymmetric plume and inserting a rectangular section of constant-property lines in between. A semiempirical scaling factor is derived to account for differences in mixing rates between axisymmetric and nonaxisymmetric plumes. The width of the rectangular section is obtained through conservation of mass. Models have been developed for 2D convergent-divergent nozzles with and without bypass air and for 2D plug nozzles. Comparisons of total-temperature predictions with test data indicate good agreement, especially in the high-temperature core region which is the major contributor to IR signature in the plume.

Nomenclature

AR = nozzle exit aspect ratio
b = width of bypass air layer
C = proportionality constant in Eq. (1)
d = nozzle height at exit
d' = exit height of basic plume with bypass flow
f = scaling factor
f_a = scaling factor at X_a
f_0 = scaling factor at the end of plug
W = width of plume midsection
W' = width of plume midsection with bypass flow
X = coordinate in nozzle axial direction
X_a = station at which two-dimensional-nozzle plume becomes axisymmetric
δ = nozzle half angle
ξ = ratio between major and minor axes

Introduction

BESIDES its advantages for vectoring and better airframe integration, the two-dimensional (2D) nozzle has been advocated as a means to lower the plume infrared (IR) signature. To evaluate and develop this concept, we need a method to predict the IR signature of a 2D-nozzle plume. Some of the available computer codes for IR signature prediction, such as General Electric's SCORPIO-N, are potentially useful for nonaxisymmetric plumes since they have the option of three-dimensional-flowfield input. The problem is hence reduced to that of obtaining three-dimensional plume flowfields for the 2D nozzle. One way of calculating such flowfields is to apply the Navier-Stokes equations. Since turbulent mixing, engine swirling, and bypass-air mixing are present and since the plumes of various 2D nozzles are highly three-dimensional, such a Navier-Stokes code would be extremely difficult to develop and expensive to run. An alternative is to model the plume approximately to provide economical and adequate plume flowfields for IR signature prediction.

Presented as Paper 80-1808 at the AIAA Aircraft Systems and Technology Meeting, Anaheim, Calif., Aug. 4-6, 1980; submitted Sept. 10, 1980; revision received June 4, 1981.

*Senior Scientist, Propulsion Research, Aircraft Division. Member AIAA.
†Senior Technical Specialist, Propulsion Research, Aircraft Division. Associate Fellow AIAA.
‡Scientist, Propulsion Research, Aircraft Division. Member AIAA.

In this paper a simple modeling technique, based on detailed flowfields of axisymmetric nozzle plumes, is derived for 2D-nozzle plumes at static conditions (i.e., exhausting into quiescent air). Subsequently, it is intended to extend the method to cover nonzero flight speeds at which the external stream affects the mixing rates and the shape of the plume.[1] Detailed constructions of various plume models for 2D convergent-divergent (2D-CD) nozzles and 2D plug nozzles will be described here in terms of proper scaling of the axisymmetric plume, application of conservation of mass, and a spreading procedure to simulate the mixing of the bypass air with the turbine exhaust and with the ambient air. A single calculation of an axisymmetric plume can provide the basic solution for a number of 2D nozzles of different configurations through a wide range of aspect ratios, with or without bypass air, as long as the nozzle pressure ratio (NPR) and throat temperature remain the same. In all, four plume examples are presented and compared with test data.[2] Two examples are given to validate the basic model for 2D-CD nozzle without bypass air. The first has a low aspect ratio of 2.53; the second, a very high aspect ratio of 52.9. Such diverse aspect ratios serve not only to verify the basic model but also to demonstrate its range of application. The modification of the basic modeling for 2D plug nozzles is illustrated by a plume example with aspect ratio 3.3, verified again by the test data.[2] The final example illustrates the modification for a 2D-CD nozzle with bypass air. A very good agreement between the prediction and test data[2] is shown.

Basic Modeling for 2D-CD Nozzle

It has been observed[3,4] that the jet issuing from a rectangular nozzle becomes approximately axisymmetric far downstream of the nozzle exit. In fact, it approaches that issuing from an equivalent round nozzle with the same momentum at the exit.[3] Consistent with this phenomenon, a plume model as illustrated in Fig. 1 is constructed for the 2D-CD nozzle. The nonaxisymmetric plume consists of two semicircular sides attached to a rectangular section in between, in accordance with experimental observations at static conditions.[1] The width of the rectangular section decreases as the plume develops so that the axisymmetric far jet is correctly reproduced. The flow properties in the semicircular sections are obtained from an axisymmetric plume, properly adjusted through a scaling factor that allows for the difference in mixing rates between the axisymmetric and nonaxisymmetric plumes. The flow properties in the midsection are assumed to be constant along straight lines connecting the two sides. The width of the midsection is determined from conservation of

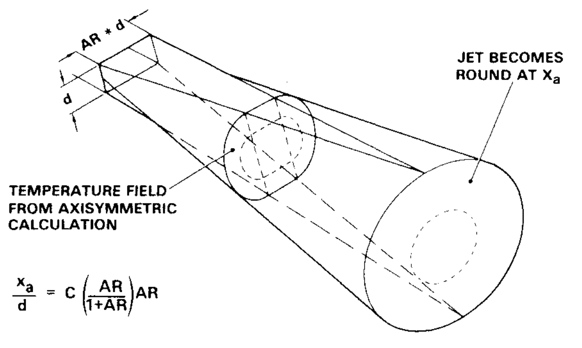


Fig. 1 Approximate calculation of temperature field.

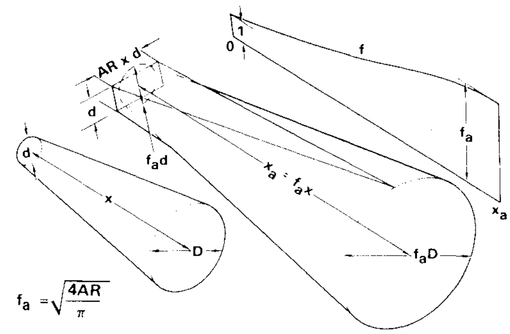


Fig. 2 Synthesis of a scaling factor.

mass. Three essential elements of the model are the distance at which the plume becomes axisymmetric, the scaling factor as a function of the axial distance, and the width of the mid-section.

Strictly, the model does not apply at or near the nozzle exit, unless the exit has truly semicircular sides. Two approaches may be taken to solve this problem approximately. The first is to alter the exit geometry to fit the model, keeping the same nozzle exit area. The second is to compute the first station at least several nozzle heights downstream of the exit and then interpolate. Here, the second approach is adopted.

Far Field

Far downstream of the exit of a 2D nozzle, the plume becomes axisymmetric. A simple formula for X_a at which the plume approaches axisymmetry is presently derived. Consider a rectangular nozzle of height d and width w at eixt. Let the width be fixed and let the height d vary through $0<d\leq w$ to change the aspect ratio. It seems reasonable to assume that X_a varies approximately with w for small d (moderate-to-high aspect ratios); hence we may write

$$X_a/w = CF$$

where C is a constant and F a function of d and w that adjusts X_a for large d (low aspect ratio down to unity). Such a function should decrease monotonically with increasing d and be bounded. The simplest appears to be $w/(w+d)$, which decreases from unity to one-half as d increases from 0 to w. Thus a simple formula for X_a is

$$X_a/w = Cw/(w+d)$$

or, in terms of the aspect ratio $\mathcal{R} = w/d \geq 1$,

$$X_a/d = C\mathcal{R}^2/(1+\mathcal{R}) \quad (1)$$

Based on test data of Refs. 3 and 4, the constant C is set to 12. From another point of view, Eq. (1) can be obtained from the assumption that the distance X_a at which the plume approaches axisymmetry is linearly proportional to the product of the hydraulic diameter and the aspect ratio at the nozzle exit. The constant C is the only experimentally determined quantity for the basic modeling technique and has remained unchanged for all the models developed so far.

Scaling Factor

In the model, flow properties of a basic axisymmetric plume are obtained either experimentally or computationally for a nozzle of exit diameter d, where d is also the height of the rectangular nozzle (Fig. 2). This plume provides the properties of the nonaxisymmetric plume, after scaling by an appropriate scaling factor. The scaling factor f is defined as the multiplier which, when applied to the radial and axial coordinates of a data point of the basic axisymmetric plume, gives the corresponding location of the nonaxisymmetric plume. A form for the variation of this scaling factor with the axial distance will now be derived.

At X_a given by Eq. (1), the plume from a 2D-CD nozzle with exit height d and aspect ratio $\mathcal{R}$ becomes axisymmetric and is the same as that from an equivalent round nozzle with the same exit momentum.[3] On the assumption of the same average momentum per unit area, the equivalent nozzle (shown by the dash line in Fig. 2) has the same exit area as the 2D-CD nozzle and hence has an exit diameter of $2\sqrt{\mathcal{R}/\pi}d$. Now, the equivalent-nozzle plume can be obtained from the basic plume through similarity, i.e., by multiplying the radial and axial coordinates of each data point of the basic plume by a factor of $2\sqrt{\mathcal{R}/\pi}$. Since within our approximation the 2D-nozzle plume and the equivalent-nozzle plume become the same at station X_a and also downstream, by definition the scaling factor at X_a is then $f=f_a=2\sqrt{\mathcal{R}/\pi}$, and it remains constant further downstream. Hence the rate of change of the scaling factor with respect to the axial distance must vanish at X_a. At the nozzle exit, the scaling factor is unity and its rate of change depends on the initial behavior of the two plumes. The cases tackled so far have all been almost perfectly expanded nozzles. Furthermore, the axisymmetric plume calculation which has been used treats the nozzle wall angle as zero. Under these circumstances the initial rate of change of the scaling factor is approximated by the tangent of the wall angle of the rectangular nozzle.§ A cubic spline, derived from two points and two slopes, relates the scaling factor f to the axial distance X (Fig. 2):¶

$$f = 1 + f_I'X + (3f_a - 2f_I'X_a - 3)(X/X_a)^2 + (2 + f_I'X_a - 2F_a)(X/X_a)^3$$

where

$$f_I' = \tan(\delta) \qquad f_a = 2\sqrt{\mathcal{R}/\pi}$$

and δ equals the nozzle half angle.

An important physical meaning of the scaling factor is that at any station with scaling factor f, the semicircular side sections of the nonaxisymmetric plume can be viewed as being a cross section of an axisymmetric plume originating from a "phantom" round nozzle with exit diameter fd. This is obviously true at X_a and downstream (see Fig. 2). To see that this is true elsewhere, we take a station with scaling factor f. By definition, the side cross section is obtained by scaling up the basic solution by f; the same cross section would be obtained if an axisymmetric plume from a round nozzle of

§This approach lets the axisymmetric calculation take care of the turbulent mixing while using the scaling factor to account for the inviscid plume development. It is preferable to the alternative approach of using the axisymmetric calculation for both turbulent mixing and inviscid development. The alternative approach would lead to a more complicated scaling factor that cannot be adequately represented by a cubic since a 2D plume develops quite differently from an axisymmetric plume.

¶In the formula X and X_a are nondimensionalized with respect to d.

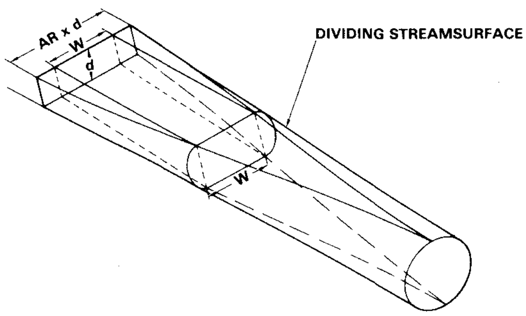


Fig. 3 Determination of plume width through conservation of mass.

diameter fd were obtained from the basic plume through similarity.

The scaling factor is derived from the cubic spline passing through two end points and assuming two end slopes. All four conditions that define the spline are based on geometric and physical considerations of the flow; the only empirical quantity X_a could be adjusted through the experimentally determined constant C in Eq. (1). The plume temperature prediction, however, appears to be rather insensitive to C since the same value of C has been found adequate for all the models developed so far. When the nozzle is perfectly or nearly perfectly expanded, the initial slope condition applies and the scaling factor f has worked satisfactorily for a wide range of nozzle geometry.

Conservation of Mass

Figure 3 shows the dividing stream surface from a 2D-CD nozzle of aspect ratio $\mathcal{R}$. Consistent with our model, a typical cross section is composed of semicircular intersections of all dividing streamlines of the axisymmetric plume on the two sides and the upper and lower connective straight lines, as shown in Fig. 3. In this section we explain how the width of the midsection is determined. Let us postulate that the portion of the plume in the midsection expands truly two-dimensionally, i.e., the mass inside the dividing stream surface per unit width of the midsection remains constant from station to station although the height of the plume increases. Meanwhile, mass is continuously being transferred from the two edges of the midsection to the semicircular sides as the width of the midsection decreases gradually to zero at X_a, where all mass has been transferred to the two semicircular sections. Now suppose we want to find the width at a certain station where the scaling factor is f. In the last section we observed that the side section is the same as an axisymmetric plume originating from a "phantom" nozzle with exit diameter fd. Now, we assume that the mass flow per unit area in the exit planes of the phantom and real nozzles is uniformly distributed: then, the mass flows emerging from these nozzles are proportional to their exit areas. It follows that the mass flow in the two semicircular sections of the three-dimensional dividing stream surface is proportional to the exit area $\pi f^2 d^2/4$ of the phantom nozzle and that the total mass flow is proportional to the exit area $\mathcal{R} d^2$ of the 2D-CD nozzle. Since the total mass flow is conserved within the dividing stream surface, the mass flow that still remains in the midsection at that station is proportional to $(\mathcal{R}-\pi f^2/4)d^2$. Since the midsection expands two-dimensionally, this mass flow is the same as the mass flow contained in a segment of width W at the nozzle exit, as represented by Wd. Equating these two, we have

$$W/d = \mathcal{R} - \pi f^2/4 \tag{2}$$

Plume Examples

The basic modeling was applied to two 2D-CD nozzles of aspect ratios 2.53 and 52.9. Both are perfectly expanded with a nozzle pressure ratio (NPR) of 3 and without bypass air

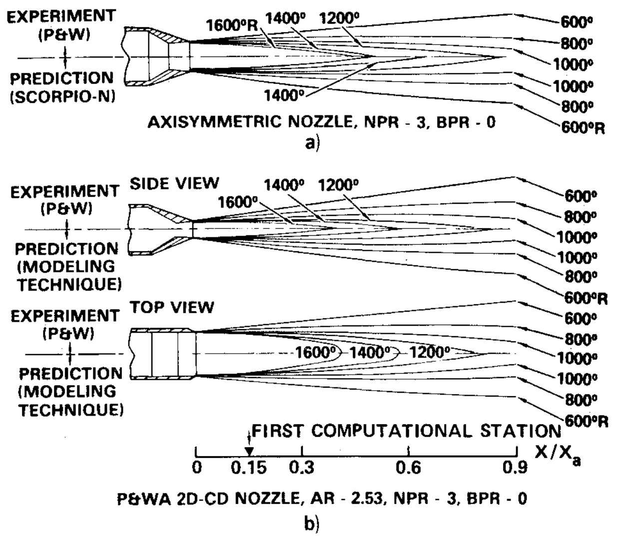


Fig. 4 Comparison of total-temperature contours for axisymmetric and low aspect ratio nozzles.

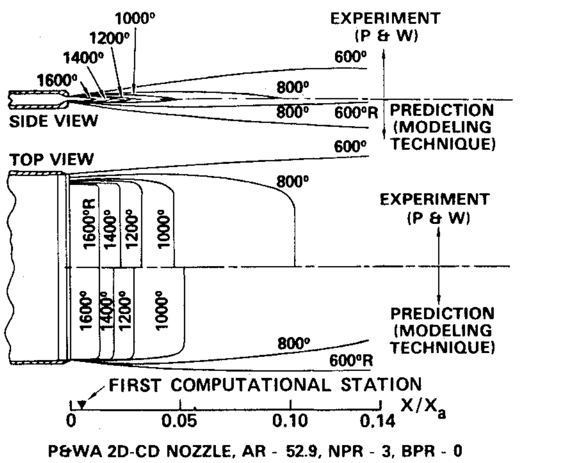


Fig. 5 Comparison of total-temperature contours for a high aspect ratio 2D-CD nozzle.

[bypass ratio (BPR) = 0]. The predicted total temperature distributions are compared with test data of Pratt & Whitney.[2] Since the test data were presented in the form of smooth curves, it was decided to present the predicted results in the lower half of each graph below the line of symmetry, while showing the test data in the upper half for comparison. The basic axisymmetric plume from a round nozzle with nozzle pressure ratio 3, exit total temperature 1760° R and exit Mach number 1.38 was computed by using the SCORPIO-N code.[5] The total-temperature contours are compared with the test data in Fig. 4a. Except for the 1400°R contour, the agreement is very good. A cross plot of the test data, however, reveals that the accuracy of the 1400°R contour is questionable since it is too close to the 1600°R contour. In Fig. 4b, predicted total-temperature contours in the plume of a Pratt & Whitney Aircraft (P&WA) 2D-CD nozzle of exit aspect ratio 2.53 are again compared with the test data. The excellent agreement,** though somewhat surprising, is not completely unexpected since a good model based on axisymmetric plumes should be expected to work well with 2D-CD nozzles of low aspect ratios. As a severe test, however, we have applied the same model to a P&WA 2D-CD nozzle of a very high exit aspect ratio of 52.9. The comparison of total-temperature contours is shown in Fig. 5. For such a nearly sheet-like

**Preliminary results show that the slight discrepancy would produce no more than 1% of error in IR signature prediction.

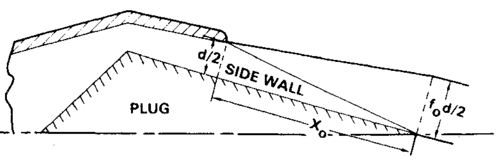


Fig. 6 Conservation of mass for 2D plug nozzle.

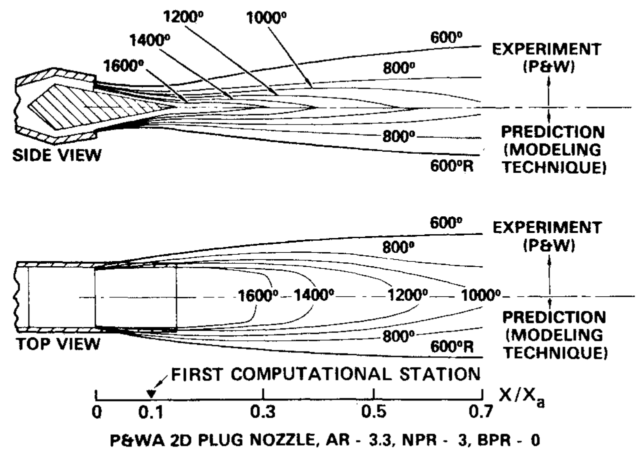


Fig. 7 Comparison of total-temperature contours for a 2D plug nozzle.

plume, mixing is predominantly two-dimensional; such a good agreement between test data and prediction by a model derived from axisymmetric plume is noteworthy. Equally remarkable is the accuracy of prediction of the plume width by the approximate mass balancing procedure. The model therefore seems to be physically realistic.

Application to 2D Plug Nozzle

If we imagine that the plug of a 2D nozzle shrinks to zero thickness, in the limit we obtain a 2D-CD nozzle. Since the mixing mechanism in the plumes of these two nozzles is similar and the development of the plume predominantly depends on mixing, the flow model is expected to apply also to 2D plug nozzles with some minor modifications.

Model Modification

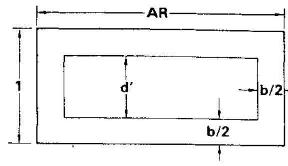
For the scaling factor, essentially the same procedure can be used. The plume is initially scaled along the plug surface up to X_0 (Fig. 6) and then in the axial direction. In the mass balancing procedure, the side walls initially inhibit mixing along the sides of the plume and so prevent some mass from being transferred to the side sections of the plume. Let f_0 be the scaling factor at X_0, at the end of the plug. Then at X_0 the mass flow that would have been transferred to the sides, if the nozzle were 2D-CD, is represented by $\pi f_0^2 d^2/4$. In the case of the plug nozzle, the side wall has a triangular area $X_0 d/4$, which tends to hold back the side spillage, while the plume up to X_0 has a trapezoidal projected area $X_0\,(1+f_0)d/4$, which avails itself to mass transfer. It is reasonable to assume that the fraction of the mass flow $\pi f_0^2 d^2/4$ that is retained in the midsection at X_0 is equal to the ratio of these two areas. Adding this retained mass to Eq. (2), we have, for 2D plug nozzles,

$$W/d = Æ - \pi f^2/4 + \pi f_0^2 (1+f_0)^{-1}/4$$

Plume Example

The modified modeling was applied to a P&WA 2D plug nozzle of exit aspect ratio 3.3. The value of the nozzle pressure ratio was again 3; the same axisymmetric plume calculation was used. The predicted total-temperature contours are presented in Fig. 7, again in comparison with P&WA test data.[2] Agreement is very good, especially of the plume width. Notice that the lengths of various temperature zones and the side view depend mainly on the scaling factor, while the widths and top view are also strongly affected by the mass balancing procedure. The slight underprediction of the length of the 1600°R zone is probably due to the absence from the model of any compensation for the slight reduction of mixing due to the side walls. On the other hand, the excellent agreement of the plume width is a further testimonial to the validity of the mass balancing procedure.

Fig. 8 Sizing of a 2D-CD nozzle with bypass air.



Modeling of a Sample 2D-CD Nozzle with Bypass Air

Modeling of plumes with bypass flow is highly complicated. The internal mixing and, to a lesser extent, the external mixing depend on the internal configurations (with or without a mixer for instance), the amount and properties of bypass air, and how and where the bypass air is introduced. Extension of the basic technique presented earlier to account for bypass flow must therefore be done to some extent on a case-by-case basis. In this section, modifications of the basic modeling for treating a P&WA 2D-CD nozzle with exit aspect ratio 2.53, nozzle pressure ratio 3, and bypass ratio (BPR) unity are presented. The purpose is to illustrate the flexibility of the modeling technique and to show how it can be adapted to more complicated cases.

For a given 2D-CD nozzle, the total exit momentum of a nonaxisymmetric plume with bypass air is less than the total exit momentum without the bypass air, if the weight flow is the same. Hence its equivalent axisymmetric plume, which it eventually approaches, is also smaller. Unless the bypass and core flows are completely mixed at exit, the presence of the bypass air will be manifested in a wider region of low temperatures in the outer regions of the plume. The velocity field for the case treated here is not available but the temperatures suggest that the bypass and core flows are only partially mixed at exit (see Fig. 11, to which further reference will be made later). From the top view, the mixing between the turbine exhaust and bypass air is seen to have proceeded to a considerable degree, yet from the side view the exhaust and bypass air appear to be hardly mixed at exit. This observation suggests that the total-temperature contours in the side view can possibly be obtained through a scaled-down basic 2D-CD plume without bypass air. The basic model is therefore modified as follows: The plume is first reduced in size by omitting the bypass flow and applying the basic model to the core flow. The effect of the bypass air is then reintroduced by stretching out the temperature contours in a manner to be described later.

A further modification to the basic model is the replacement of the semicircular shape of the side sections by semiellipses. The purpose of this is to allow for the fact that, at exit, the plume is more highly mixed across the wide dimension than it is across the narrow dimension (again, see Fig. 11).

Size Factor

Without any information about the bypass flow properties at the nozzle exit, we took the simplest approach by assuming that the exit area for the bypass air is the same as that for the turbine exhaust when BPR = 1,† †and that the thickness of the

††For BPR around unity, these areas would be in the ratio BPR:1.

bypass air layer is the same along the four nozzle walls (Fig. 8). This assumption leads to

$$(AR - b)(1 - b) = AR/2$$

the solution of which is

$$b = (AR + 1 - \sqrt{AR^2 + 1})/2$$

When the aspect ratio is 2.53, we have $b = 0.40$ and $d' = 1 - b = 0.60$.

Hence the side view of the plume can be derived from a basic plume from a 2D-CD nozzle of aspect ratio 2.53 without bypass air, provided the plume without bypass air is sized down to 60%. Once the side view of the plume is obtained, the top view can be derived from it through the inclusion of the midsection. Because of the internal mixing, however, the width of the midsection is determined by a modified mass balancing procedure as described below.

Conservation of Mass

A number of assumptions are made for the determination of the width of the midsection. The plume core, which consists of the 60% basic plume after some exchange of mass flow with the bypass air, is assumed to approach axisymmetry at the same station as the 60% basic plume; since the side view is determined by the 60% basic plume, the complete plume with bypass air approaches axisymmetry at X_a which is 60% of that for the case without bypass air. It is also assumed that the plume core dividing stream surface (see Fig. 9), within which the mass flow remains constant, is composed of a flat midsection and elliptic side sections as shown. At the nozzle exit the cross-sectional area of the dividing stream surface is set equal to that of the 60% basic plume, i.e., ARd'^2; it is assumed that the mass flows in both cases are approximately equal, so that the dividing stream surface of the plume core approaches that of the 60% basic plume when they both become axisymmetric at X_a.

Additional geometric assumptions are 1) the ratio between the major and minor axes ξ decreases from 2.53 (the nozzle-exit aspect ratio) to unity; and 2) the slope and curvature of ξ as functions of the axial distance vanish at X_a, where the plume becomes approximately axisymmetric (see Fig. 9). A cubic satisfying conditions 1 and 2

$$\xi = 1 + (AR - 1)(X_a - X)^3 / X_a^3$$

is used to interpolate for this ratio at various stations. When the balance of mass is applied to the plume core, essentially the procedure developed for 2D-CD nozzle without bypass air can be used to determine the width of the mid-flat-section. The only difference is in the mass flow of the semielliptic side sections. Take a station with scaling factor f. Let d' be the height of the scaled-down basic plume at the nozzle exit; then the mass flow of the side sections, if they were semicircular, would be represented by $\pi f^2 d'^2/4$. Since the area ratio between an ellipse and its inscribed circle is equal to ξ, the major to minor axes ratio, the mass flow in the elliptic side sections is represented by $\pi \xi f^2 d'^2/4$. Hence we obtain for the width W',

$$W'/d = AR - \pi \xi f^2/4$$

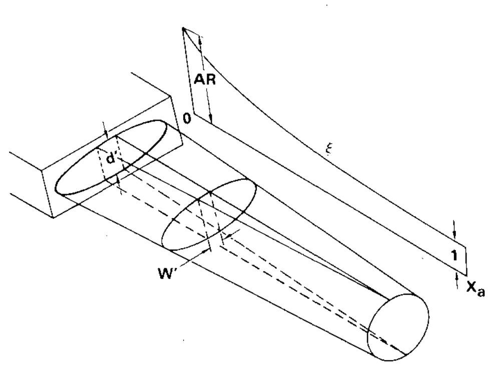


Fig. 9 Dividing stream surface for 2D-CD nozzle with bypass air.

Spreading Due to Bypass Flow

The spreading procedure is illustrated in Fig. 10. Figure 10a shows the nozzle exit and the cross section of the basic plume still with semicircular side sections; Fig. 10c shows the cross sections of the basic plume boundary and the final boundary at X_a where the plume becomes approximately axisymmetric; and Fig. 10b shows a typical intermediate cross section. All solid lines in Fig. 10b represent various total-temperature contours of the basic plume, while the outer dashed line represents the outer boundary of the complete plume after proper spreading for the bypass flow. Though specific techniques could be developed for stretching the semicircular side section to semielliptic and for simulating further mixing due to the bypass flow, it is practical to combine these two in a single spreading procedure. The spreading takes place in two stages, as illustrated in Fig. 10b. Before spreading can begin, the final positions of the plume boundary (the lowest total-temperature contour) at various stations, as indicated by the dashed lines, need to be determined. At the exit (Fig. 10a), the boundary must be raised vertically through a displacement a_0 to coincide with the nozzle surface. It is reasonable to scale up this displacement at subsequent stations by the local scaling factors, i.e., $a_1 = f_1 a_0$, $a_a = f_a a_0$, etc. By symmetry, the horizontal displacement at X_a (Fig. 10c) is equal to the vertical one, $b_a = a_a$. At the exit (Fig. 10a), the horizontal displacement is b_0. In between, it is given by a quadratic with zero slope at X_a; for example, the displacement b_1 at $X = X_1$ (Fig. 10b) is

$$b_1 = b_a + (b_0 - b_a)(X_a - X_1)^2 / X_a^2$$

The spreading through the displacement to simulate the external mixing due to the bypass flow depends on the extent of the internal mixing as manifested by the total temperature distribution at the nozzle exit. In this case, as mentioned earlier, more vigorous internal mixing takes place in the horizontal direction than in the vertical direction. Accordingly, in the spreading procedure the horizontal spreading is assumed to penetrate deeper into the plume to exclude only the innermost high-temperature contour, whereas the vertical spreading is allowed to operate only in the outer low-temperature region.

In the first stage (Fig. 10d) the spreading is made primarily for the vertical direction. Since the size factor for the basic plume is 60%, the vertical displacement a_0 in Fig. 10a is 40% of the half-nozzle height. The spreading therefore spans a range of 40%. At subsequent stations, the ranges of spreading

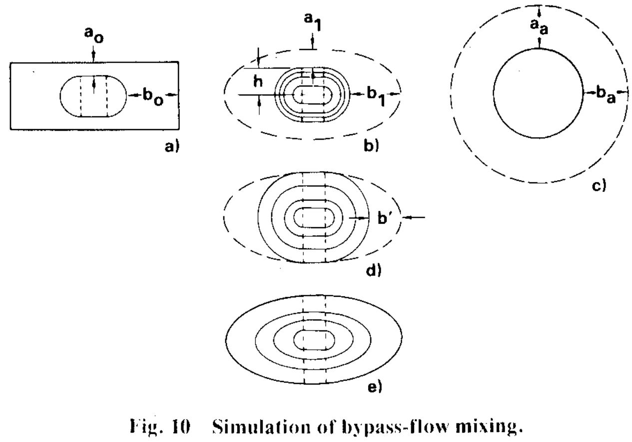


Fig. 10 Simulation of bypass-flow mixing.

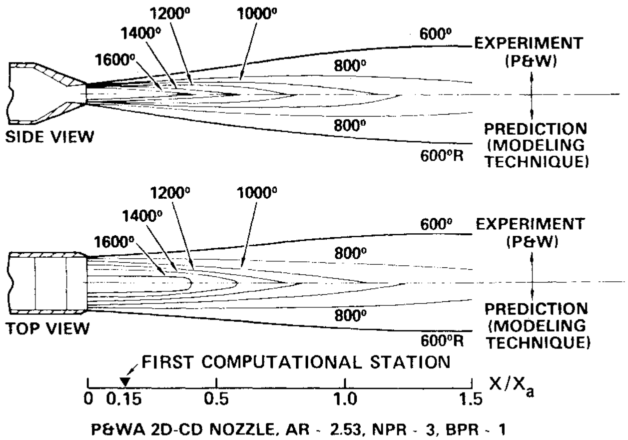


Fig. 11 Comparison of total-temperature contours for a 2D-CD nozzle with bypass air.

must be postulated. For simplicity, subsequent vertical spreadings are assumed to affect only the outer 40% of the basic plume, e.g., contours having half-heights from $0.6h$ to h in Fig. 10b. (Just for illustration, assume only the outer two contours satisfy this condition.) The vertical spreading is done through a linear stretching; the outermost contour with half-height h is moved through the maximum displacement a_l, while the contours with half-height $0.6h$ or less remain stationary. To achieve axisymmetry at X_a, a simultaneous stretching of an equal amount is applied horizontally. The resulting contours are illustrated in Fig. 10d. The final spreading extends horizontally through a maximum displacement of $b' = b_l - a_l$ (see Figs. 10b and 10d). The stretching is again linear, based on the original half-widths of the contours in the horizontal plane of symmetry. It is applied to all contours except the innermost one, which remains unchanged as shown in Fig. 10e.

The spreading at the exit station needs a special treatment. It is modeled after a station several nozzle heights downstream. Specifically, the ratio of the distance between any two adjacent contours to the distance between the highest and lowest contours is assumed to be the same, either horizontally or vertically, at both stations.

Plume Example

The modified modeling technique was applied to the plume of a P&WA 2D-CD nozzle of exit aspect ratio 2.53. The nozzle pressure ratio was 3 and the bypass ratio, unity. The predicted total-temperature contours are compared with test data[2] in Fig. 11. Though some features of the model were qualitatively influenced by test data, the agreement shown by Fig. 11 is very good.

Concluding Remarks

The simple modeling technique based on physical insight, similarity, and global flow behavior works well for 2D-CD and 2D plug nozzles at static conditions. Its predicted plume flowfields are adequate for plume IR signature analysis, as comparisons with test data show. The success thus far encourages a full development of the modeling technique as a useful tool for IR signature prediction.

Specifically, the technique is being extended‡‡ to treat 1) more general configurations, 2) more general internal conditions, for instance, complicated bypass flow, and 3) more general external conditions, such as simplified external stream or in-flight external flow. Meanwhile, the SCORPIO-N code will be used, with modifications for 2D-nozzle plumes, to predict various plume IR signatures, which in turn will be compared with available test data to validate or improve the modeling.

Acknowledgments

This work was performed under Northrop IR&D program. The authors wish to thank B.L. Hunt for discussion and encouragement and wish to note L.S. Wang's help in the conversion (from CDC to IBM) of the SCORPIO-N code.

References

[1]Kurn, A.G., "Observations of the Flow from a Rectangular Nozzle," Royal Aircraft Establishment, Tech. Rept. 74043, March 1974.

[2]Stevens, H.L. and Herrick, P.W., "Installed Turbine Engine Survivability Criteria (ITESC) Program—Semiannual Interim Technical Report No. 5," Pratt & Whitney Aircraft Group, Rept. FR-10714, Dec. 1978.

[3]Sfeir, A.A., "Investigation of Three-Dimensional Turbulent Rectangular Jets," *AIAA Journal*, Vol. 17, Oct. 1979, pp. 1055-1060.

[4]Wang, J.C., "Laser Velocimetry Measurements on High Temperature Round and Rectangular Twin Jet Flows," *Proceedings of the Fifth Biennial Symposium on Turbulence*, Science Press, Princeton, N.J., 1979, pp. 435-443.

[5]Wilton, M.E., "Advanced Infrared Signature Prediction Program, Spectral Calculation of Radiation from a Turbine Propulsion System as Intercepted by an Observer (SCORPIO-N)," Vol. III—Analysis, Contract Rept. R 78AEG314, Naval Air Propulsion Test Center, Trenton, N.J., Nov. 1979.

[6]Chu, C.-W. and Der, J. Jr., "Modeling of 2D-Nozzle Plume for IR Signature Prediction under Static Conditions," AIAA Paper 81-1108, June 1981.

‡‡The modeling technique has been extended to cover ADEN, treat more general bypass flow, and account for the effect of engine swirl (see Ref. 6).